Chapter 7
Position Control with Trajectory Profiler

Introduction

The speed controller as described in Chapter 1, uses a local observer using a simple proportional-only feedback loop with a following integrator to generate the rotor position θ'. With this arrangement, position control is easily added using a simple proportional position loop around the speed controller.

The position controller is enhanced by adding a built-in trajectory profiler, which forces the speed to follow a trapezoidal trajectory when the position reference θ^* changes too fast for the position controller to follow it. This is achieved very neatly by making the position loop gain value $K_{\theta P}$ a function of the position error $\Delta\theta$, in combination with acceleration and speed limiters.

Position Control Implementation

The addition of the position control loop is shown in Figure 7.1.

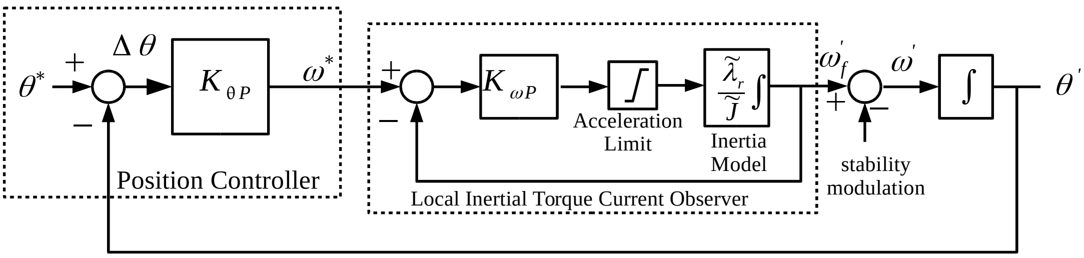

Figure 7.1. Addition of position controller.

The transfer function for the dual proportional position controller, as shown in Figure 7.5, is given by:

$$\frac{\theta'(s)}{\theta^*(s)} = \frac{K_{\theta P} K_{\omega P} \frac{\tilde{\lambda}_r}{\tilde{J}}}{s^2 + s K_{\omega P} \frac{\tilde{\lambda}_r}{\tilde{J}} + K_{\theta P} K_{\omega P} \frac{\tilde{\lambda}_r}{\tilde{J}}} \qquad (7.1)$$

The characteristic equation for this system is:

$$s^2 + s\,K_{\omega P}\frac{\tilde{\lambda}_r}{\tilde{J}} + K_{\theta P}K_{\omega P}\frac{\tilde{\lambda}_r}{\tilde{J}} = 0 \tag{7.2}$$

By comparing this with the universal characteristic equation for a second-order system with damping factor ζ and natural resonant frequency ω_0:

$$s^2 + s\,2\,\zeta\omega_0 + \omega_0^2 = 0 \tag{7.3}$$

the two proportional gain settings are:

$$K_{\theta P} = \frac{\omega_0}{2\,\zeta} \tag{7.4}$$

$$K_{\omega P} = 2\,\zeta\omega_0\frac{\tilde{J}}{\tilde{\lambda}_r} \tag{7.5}$$

Suitable per-unit parameter constants for natural frequency and damping are $K_{\theta f} = \omega_0/\omega_n$ and $K_{\theta d} = \zeta$. Using estimated values of the machine parameters, these give:

$$K_{\theta P} = \frac{K_{\theta f}\,\tilde{\omega}_n}{2\,K_{\theta d}} \tag{7.6}$$

$$K_{\omega P} = 2\,K_{\theta d}K_{\theta f}\,\tilde{\omega}_n\frac{\tilde{J}}{\tilde{\lambda}_r} \tag{7.7}$$

Default values are $K_{\theta d} = 1$ and $K_{\theta f} = 0.5$ for fast settling time without overshoot. Note that in this controller, when only the speed control is used, speed-gain constant K_ω shown in Figure 5.2 is replaced with $K_{\theta d}K_{\theta f}$.

Trajectory Profiler

In a conventional controller, a trajectory profile is pre-calculated then applied to the position reference θ^*. The most common profile used is a trapezoidal speed profile, as shown in Figure 7.2, where the motor accelerates at a constant rate until a maximum speed is reached. The speed is then maintained up to a pre-calculated point, after which the motor decelerates to standstill, which corresponds to the final position. Pre-calculating the trajectory profile has many problems. Changing the trajectory mid-flight is difficult and a temporary speed droop due to overload cannot be accommodated.

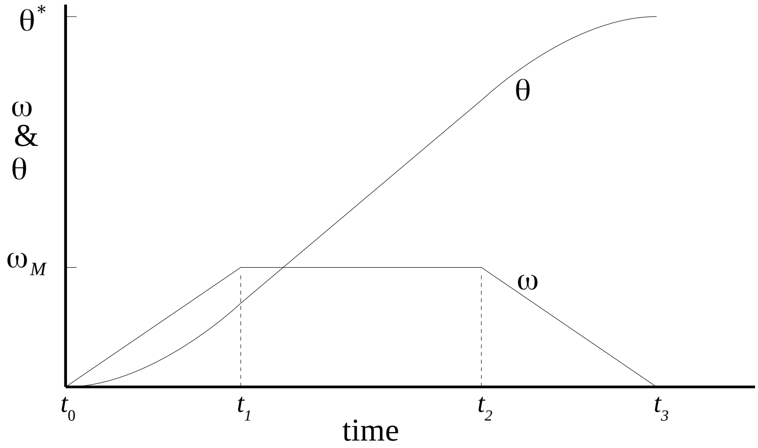

Figure 7.2. Speed and position for a trapezoidal profile.

For this controller, instead of using a profiler, the position cascaded control loops can be modified to provide the required tracking profile. To do this, use is made of the available speed and acceleration limits ω_M and A_M, and a non-linear $K_{\theta P}(\Delta\theta)$ is implemented. The position loop added to the speed controller is shown in Figure 7.3.

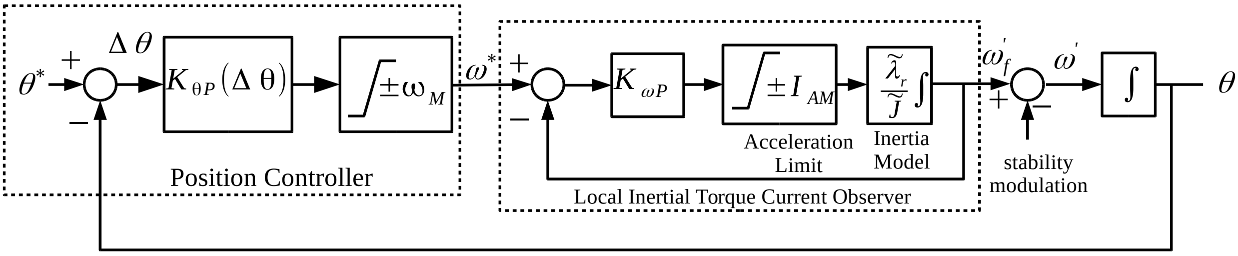

Figure 7.3. Position loop with profiler capability.

When there is a change in the position set-point, the position controller needs to produce a trapezoidal speed profile where the new set-point is reached by accelerating at a fixed rate A_M up to the maximum speed ω_M, running at this maximum speed as long as possible, and then decelerating at rate A_M, timed so that the set point position is reached at zero velocity. The graphs of the speed and position profiles for a position change from 0 to θ^* are shown in Figure 7.2.

The initial acceleration rate and the maximum speed are controlled by A_M and ω_M. Control of the point when deceleration is started, and the following deceleration rate, is achieved by a suitable choice of the non-linear function $K_{\theta P}(\Delta\theta)$. The required function $K_{\theta P}(\Delta\theta)$ can be derived as follows, with Δt being the remaining time to reach the set point position:

Assuming a deceleration rate of A_M matching the acceleration rate, the required equation of motion is:

$$\omega = A_M \Delta t \tag{7.8}$$

giving for the case of a positive position error $\Delta\theta$:

$$\Delta\theta = \frac{1}{2} A_M \Delta t^2 \\ = \frac{1}{2} \frac{\omega^2}{A_M} \tag{7.9}$$

giving:

$$\omega = \sqrt{2 A_M \Delta\theta} \tag{7.10}$$

giving the required $K_{\theta P}(\Delta\theta)$ as:

$$K_{\theta P}(\Delta\theta) = \frac{\sqrt{2 A_M \Delta\theta}}{\Delta\theta} \tag{7.11}$$

In a practical digital controller implementation, $\Delta\theta\ K_{\theta P}(\Delta\theta)$ would be calculated directly rather than calculate $K_{\theta P}(\Delta\theta)$ separately first. Note that $K_{\theta P}(\Delta\theta)$ will increase indefinitely as $\Delta\theta$ approaches zero. To maintain stability, the maximum value of $K_{\theta P}$ must be limited to that given by Equation (7.5).

Imposing this limit will result in a slightly longer settling distance for $\Delta\theta$. To compensate for this, $K_{\theta P}(\Delta\theta)$ can be adjusted to:

$$K_{\theta P}(\Delta\theta) = \frac{\sqrt{2(\Delta\theta - \theta_S) A_M}}{\Delta\theta} \quad \text{map.} \tag{7.12}$$

where θ_S is the extra settling distance.

Choose θ_S so that the maximum value of $K_{\theta P}(\Delta\theta)$ is $\omega_0/2\zeta$, the limit value of $K_{\theta P}$ from Equation (7.5). This maximum value occurs at $\Delta\theta = 2\theta_S$ and is given by:

$$\text{MAX} \quad K_{\theta P}(\Delta\theta) = \sqrt{\frac{A_M}{2\theta_S}} \tag{7.13}$$

Equating this to $\omega_0/2\zeta$ gives the required value of θ_S as:

$$\theta_S = \frac{2 A_M \zeta^2}{\omega_0^2} \tag{7.14}$$

or:

$$\theta_S = \frac{A_M}{2 K_{\theta P}^2} \tag{7.15}$$

where $K_{\theta P}$ is $\omega_0/2\zeta$.

With θ_S set to the above value, as $\Delta\theta$ drops to zero, from a positive value, $K_{\theta P}(\Delta\theta)$ can be frozen to its maximum value of $\omega_0/2\zeta$ when $\Delta\theta$ reaches $2\theta_S$ resulting in a smooth transition of the position controller to its linear mode, from where the position error will drop to zero with a response curve depending on the setting of the damping coefficient.

In summary, for a trapezoidal velocity profile, $K_{\theta P}(\Delta\theta)$ should be set as follows:

$$
\begin{aligned}
&K_{\theta P}(\Delta\theta)=\frac{\sqrt{2(|\Delta\theta|-\theta_S)A_M}}{|\Delta\theta|} && |\Delta\theta|>2\theta_S \\
&K_{\theta P}(\Delta\theta)=\frac{\omega_0}{2\zeta} && |\Delta\theta|\le 2\theta_S \\
&\text{where}\quad \theta_S=\frac{2A_M\zeta^2}{\omega_0^2}
\end{aligned}
\tag{7.16}
$$

In the actual controller, ζ and ω_0 are replaced with the estimated values $K_{\theta d}$ and $K_{\theta f}\,\tilde{\omega}_n$.

Differences to Conventional Position Controller

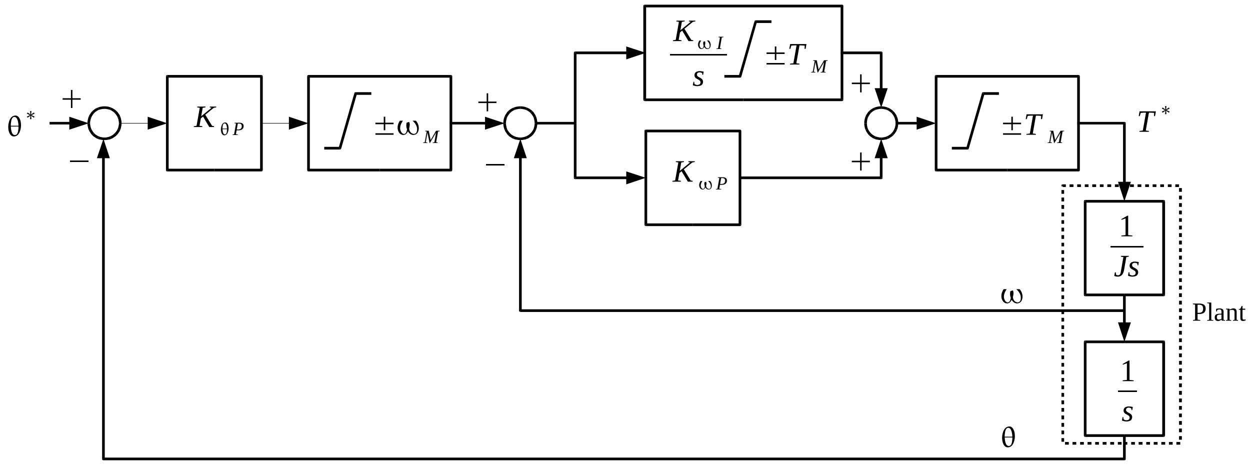

Figure 7.4 Conventional position loop structure.

Usually, position control is added by adding a proportional position loop around a proportional -integral speed control loop. The traditional control structure together with the transfer functions of the torque controlled motor drive assuming an inertial load is shown in Figure 7.4.

This control structure is a third order systems which causes several problems in use. Under severe position step inputs where the torque limit is heavily saturated, the system goes unstable. Also, tuning is difficult with three control parameters to be chosen.

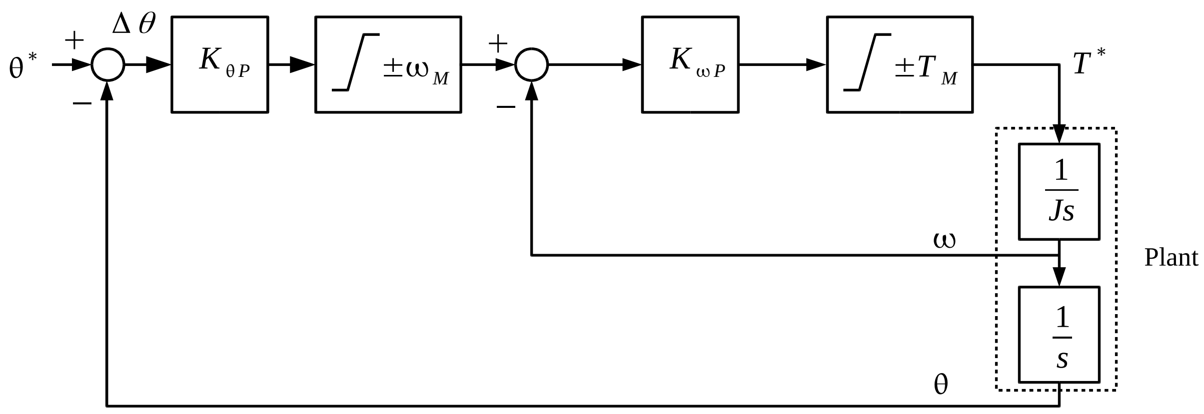

Figure 7.5. Dual proportional position loop structure.

The equivalent position control system with the present design is the much simpler and more robust dual proportional position control structure shown in Figure 7.5 (noting that the plant inertial term is modelled in software). This is a second order system which is always stable under large step inputs. It is also much easier to tune with the two control parameters dictating the second order response curve. This structure can not used in traditional controllers because a load torque offset at zero speed will cause a steady state position error.